



A Unified Framework of Fuzzy Neutrosophic Supra Set , Turiyam Set For Advanced Medical Diagnosis

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Abstract: In this paper focus of the fuzzy neutrosophic supra set is an advanced mathematical concept that merges fuzzy set theory, neutrosophic logic and supra topology to address uncertainty, vagueness and indeterminacy more effectively than classical structures. Decision algorithms based on neutrosophic supra sets can distinguish between symptoms and test results with varying certainty, improving diagnostic accuracy under uncertainty. A topological structure on fuzzy neutrosophic supra set is considered as a tool to derive some characterizations of these concept. Also we define a Fuzzy Neutrosophic Turiyam Set as an extension and apply to decision making problem. However, in highly complex real-world problems such as medical diagnosis, multi-criteria decision making, and Social Sciences- the classical neutrosophic approach still faces limitations when information extends beyond truth, indeterminacy, and falsity. To overcome this, the concept of the Turiyam set has been proposed. Analogously, a Fuzzy Neutrosophic Turiyam Set (FNTS) introduces a fourth dimension to the neutrosophic structure. Along with truth (T), indeterminacy (I), and falsity (F), the Turiyam component (Y) represents situations that transcend the known triad - capturing unknown, transcendental, or beyond-human-perception states of information. This framework is particularly powerful in medical diagnosis, where certain symptoms and disease relations cannot be explained merely by true/false or uncertain values, but require a fourth dimension to represent “unexplored” or “beyond - known” medical states.

Keywords: Fuzzy neutrosophic supra set, fuzzy decision making, Medical diagnosis under uncertainty, Indeterminacy modeling and knowledge representation, fuzzy neutrosophic turiyam set.

1. Introduction

The majority of problems in engineering, medical research, economics, and the environment and fraught with uncertainty. Fuzzy sets were introduced by L.A.Zadeh [8] in 1965. Florentine Smarandache [4] developed neutrosophic set and logic of a generalization of the intuitionistic fuzzy logic and set respectively. In Mashhour et al. [6] introduced the concepts of supra topological spaces, supra open sets and supra closed sets. Later on ME Abd El-Monsef et al.[1] introduced the concept of fuzzy supra topological spaces. A.A. Salama and S.A. Alblowi [7] introduced and studied

neutrosophic topological spaces. The fuzzy neutrosophic supra set is used manage uncertainty using the truth, indeterminacy and falsity grades of membership which are independently considered. Fuzzy neutrosophic supra sets are being used in decision making, medical diagnosis and optimization problems.

The sets T,I,F are not necessarily intervals but may be any real sub-unitary subsets of $]0,1[$. The neutrosophic components T,I,F represents the truth value, indeterminacy value and falsehood value respectively. In fuzzy neutrosophic set, indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations, and neutrosophic set by its virtue of handling inconsistent and incomplete information seems to be a better choice for medical knowledge base, because it is often not possible to have 100% of information at hand while making decision. We consider the fuzzy neutrosophic set which takes the value from the subset of $[0,1]$. In this paper, we define the fuzzy neutrosophic supra set in Medical Diagnosis under uncertainty. Also we have discussed the notion of fuzzy neutrosophic turiyam set, generalizes neutrosophic logic by introducing a fourth parameter- the turiyam value - to model transcendental or unexplained uncertainty.

2. Preliminaries

Here we recall the concept of fuzzy neutrosophic basic definitions.

Definition 2.1 [5]. Let X be a non-empty set and P is a fuzzy neutrosophic set (FNS) is an object having the form $P = \{\langle y, T_p(y), I_p(y), F_p(y) \rangle : y \in X\}$ where the functions

$T_p : X \rightarrow]0,1[$, $I_p : X \rightarrow]0,1[$, $F_p : X \rightarrow]0,1[$ denote the degree of membership function (

namely $T_p(y)$), the degree of indeterminacy function (namely $I_p(y)$), and the degree of non

membership (namely $F_p(y)$) respectively of each element $y \in X$ to the set P and

$^+0 \leq T_p(y) \leq I_p(y) \leq F_p(y) \leq 1^+$, for each $y \in X$.

Remark 2.2 [5]. Every fuzzy set P on a non-empty set X is obviously a FNS having the form

$P = \{\langle y, T_p(y), I_p(y), F_p(y) \rangle : y \in X\}$ and $R = \{\langle y, T_R(y), I_R(y), F_R(y) \rangle : y \in X\}$.

We consider two definitions for subsets ($P \subseteq R$):

$P \subseteq R \Leftrightarrow T_p(y) \leq T_R(y), I_p(y) \leq I_R(y), F_p(y) \geq F_R(y) \text{ for all } y \in X,$

$P \subseteq R \Leftrightarrow T_p(y) \leq T_R(y), I_p(y) \geq I_R(y), F_p(y) \geq F_R(y) \text{ for all } y \in X.$

Definition 2.3 [7]. Let $\{P_j : j \in J\}$ be an arbitrary family of FNSs in X , where

$P_j = \{\langle y, T_{p_j}(y), I_{p_j}(y), F_{p_j}(y) \rangle : y \in X\}$ then

$\cap P_j$ may be defined as two types:

$$\begin{aligned}
1. \cap P_j &= \{\langle y, \wedge_{j \in J} T_{p_j}(y), \wedge_{j \in J} I_{p_j}(y), \vee_{j \in J} F_{p_j}(y) \rangle : y \in X\} \\
\cap P_j &= \{\langle y, \wedge_{j \in J} T_{p_j}(y), \wedge_{j \in J} I_{p_j}(y), \vee_{j \in J} F_{p_j}(y) \rangle : y \in X\} \\
2. \cup P_j &= \{\langle y, \vee_{j \in J} T_{p_j}(y), \vee_{j \in J} I_{p_j}(y), \wedge_{j \in J} F_{p_j}(y) \rangle : y \in X\} \\
\cup P_j &= \{\langle y, \vee_{j \in J} T_{p_j}(y), \vee_{j \in J} I_{p_j}(y), \wedge_{j \in J} F_{p_j}(y) \rangle : y \in X\}
\end{aligned}$$

Definition 2.4 [7]. Let $P = \{\langle y, T_p(y), I_p(y), F_p(y) \rangle : y \in X\}$ be a FNSs in X , then the complement of the set $P(\text{CO}(P))$ may be defined as three kinds of complements.

$$\begin{aligned}
1. \text{CO}(P) &= \{\langle y, F_p(y), I_p(y), T_p(y) \rangle : y \in X\} \\
2. \text{CO}(P) &= \{\langle y, F_p(y), 1 - I_p(y), T_p(y) \rangle : y \in X\} \\
3. \text{CO}(P) &= \{\langle y, 1 - T_p(y), 1 - I_p(y), 1 - F_p(y) \rangle : y \in X\}
\end{aligned}$$

Definition 2.5 [5]. Let X be a non-empty set and the FNSs P and R be in the form

$$P = \{\langle y, T_p(y), I_p(y), F_p(y) \rangle : y \in X\} \text{ and } R = \{\langle y, T_R(y), I_R(y), F_R(y) \rangle : y \in X\} \text{ on } X$$

$$\begin{aligned}
1. P &= R \Leftrightarrow P \subseteq R \text{ and } R \subseteq P. \\
2. P - R &= \{\langle y, T_p(y) \wedge F_R(y), I_p(y) \wedge (1 - I_R(y)), F_p(y) \vee T_R(y) \rangle : y \in X\} \\
3. [P] &= \{\langle y, T_p(y), I_p(y), 1 - T_p(y) \rangle : y \in X\} \\
4. \langle P &= \{\langle y, 1 - T_p(y), I_p(y), F_p(y) \rangle : y \in X\} \\
5. O_N &= \langle y, 0, 0, 1 \rangle \text{ and } 1_N = \langle y, 1, 1, 0 \rangle.
\end{aligned}$$

Definition 2.6 [2]. A fuzzy neutrosophic supra topology (FNST) a non-empty set X is a family τ^μ of fuzzy neutrosophic supra subsets in X satisfying the following axioms.

$$\begin{aligned}
(a) 0_N, 1_N &\in \tau^\mu \\
(b) \cup_{G_i} &\in \tau^\mu, \forall \{G_i : i \in J\} \subseteq \tau^\mu
\end{aligned}$$

In this case the pair (X, τ^μ) is called a Fuzzy neutrosophic supra topological space (FNSTS) and any fuzzy neutrosophic supra set in τ^μ is known as fuzzy neutrosophic supra open set (FNSOS) in X . The element of τ^μ are called open fuzzy neutrosophic supra sets.

The complement of FNSOS in the FNSTS (X, τ^μ) is called fuzzy neutrosophic supra closed set (FNSCS).

Definition 2.7 [2]. The complement of $P(\text{CO}(P))$ of FNSOS P is called a FNCS in X .

Definition 2.8 [2]. Let (X, τ^μ) be FNSTS and $P = \langle y, T_p, I_p, F_p \rangle$ be a FNSS in X . Then the fuzzy neutrosophic supra interior (FNSI) and fuzzy neutrosophic supra closure (FNCS) of P are defined by

$$\begin{aligned}
\text{FNScl}(P) &= \cap \{Q : Q \text{ is a FNCS in } X \text{ and } P \subseteq Q\}, \\
\text{FNSint}(P) &= \cup \{R : R \text{ is a FNSOS in } X \text{ and } R \subseteq P\}.
\end{aligned}$$

Now that $\text{cl}(P)$ is a FNSCS & $\text{int}(P)$ is a FNSOS in X . Further,

1. P is a FNSCS in X iff $\text{cl}(P) = P$
2. P is a FNSOS in X iff $\text{int}(P) = P$.

3. Relations on Fuzzy Neutrosophic Supra Set

Definition 3.1: Let A be a fuzzy neutrosophic supra set in (X, T^*) and B be a fuzzy neutrosophic supra set in (Y, T^*) in a fuzzy neutrosophic supra topological space from X to Y . Then max-min-max composition of fuzzy neutrosophic supra set with A is another fuzzy neutrosophic supra set B of Y which is denoted by $R \circ A$. Then the membership function, indeterminate function

$$T_{R \circ A}(y) = \vee [T_A(x) \wedge T_A(x, y)],$$

and non-membership function of B is defined by $I_{R \circ A}(y) = \vee [I_A(x) \wedge I_A(x, y)]$ and

$$F_{R \circ A}(y) = \wedge [F_A(x) \vee F_A(x, y)], \forall y \in Y.$$

Definition 3.2: Let A be a fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X, T^*) . Then the value function of A is defined as $V^*(A) = T_A + (1 - I_A) - F_A$, where

T_A, I_A and F_A denote the truth value, indeterministic value and false value of A respectively.

Definition 3.3: Let A and B be two fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X, T^*) . Then the score function of A and B defined as $S_1^* = V^*(A) - V^*(B)$.

Definition 3.4: Let A and B be two fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X, T^*) . Then the score function of A is defined as $S_2^* = T_j - I_j \cdot F_j$.

4. Methodology and Algorithm

In this section we present a method for medical diagnosis using fuzzy neutrosophic supra sets in a fuzzy neutrosophic supra topological space (X, T^*) . Assume that there is a set of patients P with a set of symptoms S related to a set of diseases D .

We apply fuzzy neutrosophic supra set theory to develop a technique for diagnosis which patient is suffering from disease.

Algorithm

Step 1: The symptoms of the patients are given to obtain the patient-symptom relation Q and are noted in Table I.

Step 2 : The medical knowledge relating the symptoms with the set of disease under consideration are given to obtain the symptom-disease relation R and are noted in Table II.

Step 3 : The composition T of the relation of patients and diseases is found using the definition 3.1., and is noted Table III.

Step 4 : The complement of Table 1 is obtained and is noted in Table IV.

Step 5 : The complement of Table 2 is obtained and is noted in Table V.

Step 6 : For the values of Table IV and Table V, the definition 3.1 is applied, and is noted in Table VI.

Step 7 : The value function is calculated for Table III and Table VI, and is given in Table VII and Table VIII respectively.

Step 8 : The score function for the values in Table VII and Table VIII is found using definition 3.3, and is noted in Table IX.

Step 9 : Another score function for the Table III is applied using the definition 3.4, and it is given Table X.

Step 10 : Find the higher score for possibility of the patient affected with the respective disease.

5. Case study

There are four patients who appealed to a medical center with symptoms fever, cough, breathlessness problem. Let the possible diseases relating to these symptoms be Covid-19 (d_1), Dengue (d_2), Tuberculosis (d_3). Now take $P = \{p_1, p_2, p_3, p_4\}$ as the universal set where p_1, p_2, p_3 and p_4 represents patients. Next consider the set $S = \{s_1, s_2, s_3\}$ as universal set where s_1, s_2, s_3 represents symptoms fever, cough, breathlessness problem respectively and the set $D = \{d_1, d_2, d_3\}$, where d_1, d_2 and d_3 represents the diseases Covid-19, Dengue and Tuberculosis respectively..

By using our data, we construct patient-symptom relation and symptom-disease relation as follows:

$$F^*(p_1) = \{s_1 / (0.9, 0.1, 0.05), s_2 / (0.6, 0.2, 0.2), s_3 / (0.4, 0.3, 0.3)\}$$

$$F^*(p_2) = \{s_1 / (0.5, 0.3, 0.2), s_2 / (0.9, 0.1, 0.1), s_3 / (0.8, 0.1, 0.1)\}$$

$$F^*(p_3) = \{s_1 / (0.85, 0.05, 0.1), s_2 / (0.8, 0.15, 0.1), s_3 / (0.9, 0.1, 0.05)\}$$

$$F^*(p_4) = \{s_1 / (0.6, 0.2, 0.2), s_2 / (0.4, 0.3, 0.3), s_3 / (0.6, 0.25, 0.25)\}$$

Then the fuzzy neutrosophic supra set $F^*(P_i)$ is a parametrized family of all fuzzy subsets over S and gives a collection of approximate description of the patient symptoms in the medical center. This fuzzy neutrosophic supra set $F^*(P_i)$ represents the patient-symptom relation Q and is given by

Table I

Q	Fever	Cough	Breathlessness
Patient 1	(0.9,0.1,0.05)	(0.6,0.2,0.2)	(0.4,0.3,0.3)
Patient 2	(0.5,0.3,0.2)	(0.9,0.1,0.1)	(0.8,0.1,0.1)
Patient 3	(0.85,0.05,0.1)	(0.8,0.15,0.1)	(0.9,0.1,0.05)
Patient 4	(0.6,0.2,0.2)	(0.4,0.3,0.3)	(0.6,0.25,0.25)

Next,

$$G^*(s_1) = \{d_1 / (0.9, 0.05, 0.05), d_2 / (0.85, 0.1, 0.05), d_3 / (0.7, 0.2, 0.1)\}$$

$$G^*(s_2) = \{d_1 / (0.85, 0.1, 0.05), d_2 / (0.6, 0.3, 0.1), d_3 / (0.65, 0.25, 0.1)\}$$

$$G^*(s_3) = \{d_1 / (0.9, 0.05, 0.05), d_2 / (0.4, 0.3, 0.3), d_3 / (0.9, 0.1, 0.1)\}$$

Then the fuzzy neutrosophic supra set $G^*(S_i)$ is parametrized family $\{G^*(s_1), G^*(s_2), G^*(s_3)\}$ of all fuzzy subsets over the set S where $G^*: S \rightarrow F_N^*(D)$ and is determined from expert medical documentation. Thus the fuzzy neutrosophic supra set $G^*(S_i)$ given an approximate description of the three diseases and their symptoms. This fuzzy neutrosophic supra set is represented by a relation matrix (symptom-disease) R and is given by

Table II

R	Covid-19	Dengue	Tuberculosis
Fever	(0.9,0.05,0.05)	(0.85,0.1,0.05)	(0.7,0.2,0.1)
Cough	(0.85,0.1,0.05)	(0.6,0.3,0.1)	(0.65,0.25,0.1)
Breathlessness	(0.9,0.05,0.05)	(0.4,0.3,0.3)	(0.9,0.1,0.1)

Then performing the transformation operation $Q \circ R$ we get the patient-disease relation T as

Table III

T	Covid-19	Dengue	Tuberculosis
Patient 1	(0.9,0.1,0.05)	(0.85,0.3,0.05)	(0.7,0.2,0.1)
Patient 2	(0.85,0.1,0.1)	(0.6,0.1,0.1)	(0.8,0.2,0.1)
Patient 3	(0.9,0.15,0.05)	(0.8,0.15,0.1)	(0.85,0.15,0.1)
Patient 4	(0.6,0.25,0.2)	(0.6,0.25,0.2)	(0.6,0.25,0.2)

From the complement of Q and R we get

Table IV

Q^1	Fever	Cough	Breathlessness
Patient 1	(0.05,0.9,0.9)	(0.2,0.8,0.4)	(0.3,0.7,0.4)
Patient 2	(0.2,0.7,0.5)	(0.1,0.9,0.9)	(0.1,0.9,0.8)
Patient 3	(0.1,0.95,0.85)	(0.1,0.85,0.2)	(0.05,0.9,0.9)
Patient 4	(0.2,0.8,0.6)	(0.3,0.7,0.4)	(0.25,0.75,0.6)

Table V

R^1	Covid-19	Dengue	Tuberculosis
Fever	(0.05,0.95,0.9)	(0.05,0.9,0.85)	(0.1,0.8,0.7)
Cough	(0.05,0.9,0.85)	(0.1,0.7,0.6)	(0.1,0.75,0.65)
Breathlessness	(0.05,0.95,0.9)	(0.3,0.7,0.4)	(0.1,0.9,0.9)

The composition values of Table IV and Table V are calculated as

Table VI

T^1	Covid-19	Dengue	Tuberculosis
Patient 1	(0.05,0.9,0.85)	(0.3,0.9,0.4)	(0.1,0.8,0.65)
Patient 2	(0.05,0.9,0.8)	(0.1,0.7,0.8)	(0.1,0.8,0.7)
Patient 3	(0.05,0.95,0.85)	(0.1,0.9,0.6)	(0.1,0.9,0.65)
Patient 4	(0.05,0.8,0.85)	(0.1,0.8,0.6)	(0.1,0.8,0.65)

The value function for Table III and Table VI is calculated as

Table VII

Value function	Covid-19	Dengue	Tuberculosis
Patient 1	1.75	1.5	1.4
Patient 2	1.65	1.4	1.5
Patient 3	1.7	1.55	1.6
Patient 4	1.15	1.15	1.1

Table VIII

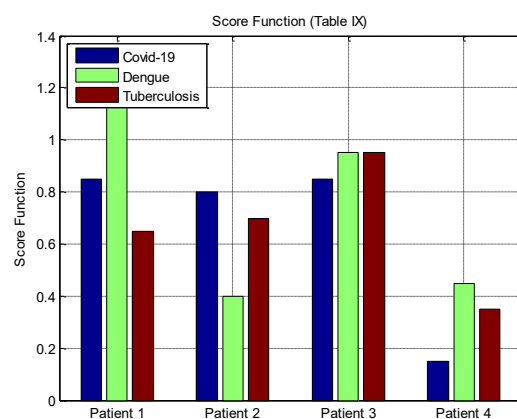
Value function	Covid-19	Dengue	Tuberculosis
Patient 1	0.9	0.2	0.75
Patient 2	0.85	1	0.8
Patient 3	0.85	0.6	0.65
Patient 4	1	0.7	0.75

Score function for the values in Table VII and Table VIII is calculated as

Table IX

Score function	Covid-19	Dengue	Tuberculosis
Patient 1	0.85	1.3	0.65
Patient 2	0.8	0.4	0.7
Patient 3	0.85	0.95	0.95
Patient 4	0.15	0.45	0.35

Figure 1 : Score Function 1

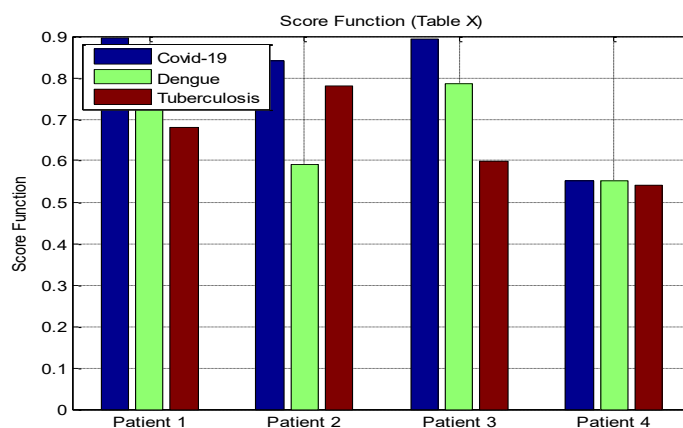


Another score function for the table III is given as

Table X

Score function	Covid-19	Dengue	Tuberculosis
Patient 1	0.895	0.835	0.68
Patient 2	0.84	0.59	0.78
Patient 3	0.893	0.785	0.6
Patient 4	0.55	0.55	0.54

Figure 2 : Score Function 2



Conclusion : It is clear from Table IX and Table X that Patient 1, 2 and 3 are suffering from Covid-19, and patient 4 is suffering from Covid -19 or Dengue.

6. Fuzzy Neutrosophic Turiyam Set & Comparison of sets

6.1 Fuzzy Neutrosophic Turiyam Set (FNTS)

Definition 6.1.1: Let X be a universal set. A Fuzzy Neutrosophic Turiyam Set (FNTS) A on X is defined as a set of ordered quadruples. $A = \{ \langle x, T_A(x), I_A(x), F_A(x), Y_A(x) : x \in X \rangle \}$ where for each element $x \in X$.

* $T_A(x) \in [0,1] \rightarrow$ degree of truth-membership

* $I_A(x) \in [0,1] \rightarrow$ degree of indeterminacy

* $F_A(x) \in [0,1] \rightarrow$ degree of falsity-membership

* $Y_A(x) \in [0,1] \rightarrow$ degree of the Turiyam

In general, $0 \leq T_A(x), I_A(x), F_A(x), Y_A(x) \leq 1$.

Remark 6.1.2:

If $T_A(x)$ is high $\rightarrow$ strong evidence for membership

If $F_A(x)$ is high $\rightarrow$ strong evidence against membership

If $I_A(x)$ is high $\rightarrow$ uncertainty or conflicting information

If $Y_A(x)$ is high $\rightarrow$ there are hidden or unknown influences (intuitional or not yet scientifically observed).

6.2 Case Study:

Application of Fuzzy Neutrosophic Turiyam Set in Diagnosing Heart Disease, Anemia and Asthma.

Patient Case :

A 52-year old patient reports to the hospital with the following major symptoms:

S_1 : Chest pain ; S_2 : Fatigue ; S_3 : shortness of Breath

The consulting physician suspects the following possible diseases:

D_1 : Heart Disease ; D_2 : Anemia; D_3 : Asthma

Because symptoms overlap across multiple conditions, an FNTS-based model is applied to improve diagnostic decision-making.

Step 1: Each symptom-disease relation is expressed as a (T, I, F, Y) and noted table I.

Table I

Symptom $\rightarrow$ Disease $\downarrow$	Heart Disease (D_1)	Anemia (D_2)	Asthma (D_3)

Chest pain (S_1)	(0.85,0.1,0.03,0.02)	(0.4,0.3,0.2,0.1)	(0.3,0.25,0.35,0.1)
Fatigue (S_2)	(0.6,0.2,0.1,0.1)	(0.8,0.1,0.05,0.05)	(0.45,0.3,0.15,0.1)
Shortness of Breath (S_3)	(0.7,0.15,0.1,0.05)	(0.5,0.25,0.15,0.1)	(0.9,0.05,0.03,0.02)

Step 2: Aggregation method

$$T(D_j) = \frac{\sum_{k=1}^m T(S_k, D_j)}{m}$$

$$F(D_j) = \frac{\sum_{k=1}^m F(S_k, D_j)}{m}$$

$$I(D_j) = \frac{\sum_{k=1}^m I(S_k, D_j)}{m}$$

$$Y(D_j) = \frac{\sum_{k=1}^m Y(S_k, D_j)}{m}$$

Where $m = 3$ symptoms.

Step 3: Aggregation for each Disease:

Heart Disease (D_1) : (0.717,0.150,0.077,0.057)

Anemia (D_2) : (0.566,0.217,0.133,0.083)

Asthma (D_3) : (0.550,0.200,0.177,0.073)

Step 4: Decision Analysis

Heart Disease (D_1) : High truth-membership (0.717), low falsity (0.077).

Anemia (D_2) : Moderate truth membership (0.566), higher indeterminacy (0.217).

Asthma (D_3) : Lower truth-membership (0.550), higher falsity (0.177).

Thus, **Heart Disease** is most probable diagnosis.

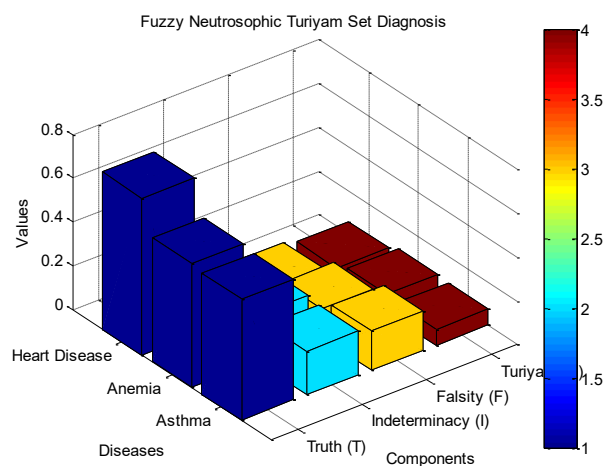
The Turiyam values (0.057,0.083,0.073) indicate the possibility of hidden/unknown causes that must be clinically investigated.

Case study conclusion

By applying the Fuzzy Neutrosophic Turiyam Set (FNTS), the physician concludes that the patient is most likely suffering from **Heart Disease (T=0.717)**.

The Turiyam component provides an additional layer of safety by alerting doctors to remain cautious about hidden factors, which is not possible in classical fuzzy or neutrosophic diagnosis models.

Figure 3: Fuzzy Neutrosophic Turiyam Set Diagnosis in Medical



6.3 Comparison of sets

Type of set	Components	Meaning	Limitation
Fuzzy set (Zadeh, 1965)	$T(x)$	Only degree of truth (membership)	Cannot handle uncertainty or falsity
Intuitionistic fuzzy set (Atanassov,1986) [3]	$T(x), F(x)$	Truth + Falsity, with constraint $T + F \leq 1$	No separate indeterminacy
Neutrosophic set (Smarandache,1999)	$T(x), I(x), F(x)$	Truth, indeterminacy, falsity independently	Hidden/unknown factors not captured.
Fuzzy Neutrosophic Turiyam Set (FNTS) (Poongothai, 2025)	$T(x), I(x), F(x), Y(x)$	Adds Turiyam (Y) to represent hidden unknown, or intuitional factors beyond logic.	Newest extension -more complex, requires new aggregation methods.

6.4 Comparative case study of Fuzzy, Intuitionistic Fuzzy, Neutrosophic and Fuzzy Neutrosophic Turiyam Sets in Medical Diagnosis

Diagnosis of Dengue with Fever & Headache

Model	Fever Values	Headache Values	Aggregate Result	Interpretation
Fuzzy Set (FS)	T = 0.8	T = 0.6	$T = \frac{0.8 + 0.6}{2} = 0.7$	70% chance of Dengue only

				truth considered.
Intuitionistic Fuzzy Set (IFS)	(T=0.8,F= 0.1)	(T=0.6,F=0.2)	$T = \frac{0.8 + 0.6}{2} = 0.7$ $F = \frac{0.1 + 0.2}{2} = 0.15$	Truth 70%, Falsity 15% remaining 15% uncertainty.
Neutrosophic Set (NS)	(T=0.8,I=0.05, F=0.1)	(T=0.6,I=0.1, F=0.15)	T=0.7,I=0.1,F=0.15	Considers truth, falsity and explicit indeterminacy
Fuzzy Neutrosophic Turiyam Set (FNTS)	(T=0.8,I=0.05, F=0.1, U=0.05)	(T=0.6,I=0.15, F=0.2, U=0.05)	T=0.7,I=0.1,F=0.15,U=0.05	Adds hidden/unknown Turiyam factor for realistic diagnosis.

7.Conclusion

In Medical diagnosis, uncertainty arises due to overlapping symptoms, incomplete patient data, and hidden risk factors. Both Fuzzy Neutrosophic Supra Set (FNSS) and Fuzzy Neutrosophic Turiyam Set (FNTS) provide powerful mathematical tools to handle such uncertainty. Thus, which FNSS enhances diagnosis by handling supra-relations among symptoms and diseases, FNTS enriches the diagnosis process by capturing Truth (T), Indeterminacy (I), Falsity (F), and hidden/unknown (Y) dimensions simultaneously. Applications of fuzzy Neutrosophic Supra Set and Fuzzy Neutrosophic Turiyam Set are especially in medical diagnosis.

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Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

References

- [1] Abd.El-Monsef M.E.and A.E.Ramadan,; On fuzzy supra topological spaces, Indian J.Pure Appl.Math. (1987); 18(4), pp. 322-329.
- [2] Amarendra Babu V and Aswini J,; Fuzzy Neutrosophic Supra Topological Spaces, Advances and Applications in Mathematical Sciences,(2021) ; 20(8), pp. 1339-1347.
- [3] Atanassov KT,; Intuitionistic fuzzy sets, Fuzzy Sets and Systems,(1986); 20, pp. 87-96.

- [4] Florentine Smarandache;; Neutrosophic logic - a generalization of the Intuitionistic fuzzy logic, Multispace and Multistructure Neutrosophic Transdisciplinarity, (2010); ISBN 978-952-67349-2-7, Vol.4, pp. 396-502.
- [5] Florentine Smarandache , Neutrosophic set - a generalization of the Intuitionistic fuzzy MMNT, (2010);ISBN 978-952-67349-2-7, Vol.4, pp. 403-409.
- [6] Mashhour AS, et al , ; On Supra topological space, Indian J.Pure Appl. Math.(1983);14(4), 502-510.
- [7] Salama AA and Alblowi SA , ; Neutrosophic set and neutrosophic topological spaces, IOSR Journal of Mathematics,(2012); 3(4), pp. 31-35.
- [8] Zadeh L A Fuzzy Sets , Inf. Control, (1965); 8(3), pp. 338-353.

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